

Transition of the dark energy equation of state in an interacting holographic dark energy model

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Abstract

A model of holographic dark energy with an interaction with matter fields has been investigated. Choosing the future event horizon as an IR cutoff, we have shown that the ratio of energy densities can vary with time. With the interaction between the two different constituents of the universe, we observed the evolution of the universe, from early deceleration to late time acceleration. In addition, we have found that such an interacting dark energy model can accommodate a transition of the dark energy from a normal state where $w_D > -1$ to $w_D < -1$ phantom regimes. Implications of interacting dark energy model for the observation of dark energy transition has been discussed.

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The present acceleration of the universe expansion has been well established through numerous and complementary cosmological observations [1]. A consistent picture has indicated that nearly three quarters of our universe consists of a “dark energy”, which is responsible for the accelerated expansion. However, the nature of such a dark energy is still rather uncertain. Explanations have been sought within a wide range of physical phenomena, including a cosmological constant, exotic fields, a new form of the gravitational equation, new geometric structures of spacetime, etc [2, 3]. Recently, a new model stimulated by the holographic principle has been put forward to explain the dark energy [4]. According to the holographic principle, the number of degrees of freedom of a physical system scales with the area of its boundary. In this context, Cohen et al [5] suggested that in quantum field theory a short distance cutoff is related to a long distance cutoff due to the limit set by formation of a black hole, which results in an upper bound on the zero-point energy density. In line with this suggestion, Hsu and Li [6][4] argued that this energy density could be viewed as the holographic dark energy density satisfying $\rho_D = 3c^2/L^2$, where c^2 is a constant, L is an IR cutoff in units $M_p^2 = 1$. Li [4] discussed three choices for the length scale L which is supposed to provide the IR cutoff, such as the Hubble radius, the particle horizon and the event horizon. He demonstrated that only identifying L with the radius of the future event horizon, we can get the correct equation of state of dark energy and obtain the desired accelerating universe. Besides other applications of the holographic principle in cosmology [7], the holographic dark energy model is a new example showing that holography is an effective way to investigate cosmology. Related works on the holographic dark energy can be found in [8]. The holographic dark energy model is found in consistent with the observational data [9].

Available models of dark energy differ in the equation of state parameter w_D as well as the variation of w_D itself during the evolution of the universe. The cosmological constant with $w_D = -1$, is located at a central position among dark energy models both in theoretical investigation and in data analysis [10]. In quintessence, K-essence, Chaplygin gas and holographic dark energy models [2, 4], w_D stays bigger than -1 , and cannot cross -1 . The phantom models of dark energy have $w_D < -1$ [3, 11]. Recently, the analysis of the type Ia supernova data indicates that the time varying dark energy gives a better fit than a cosmological constant [12]. These analysis mildly favor the evolution of the dark energy parameter w_D from $w_D > -1$ to $w_D < -1$ at recent stage. Although the galaxy cluster gas mass fraction data do not support the time-varying w_D [13], theoretical attempts toward the understanding of the w_D crossing -1 phenomenon have been started. Some dark energy models, such as the one containing a negative kinetic scalar field and a normal scalar field [14], or a single scalar field model [15], a Gauss-Bonnet brane world with

induced gravity [16] and the generalized implicitly defined dark energy equation of state model [17], have been constructed to gain insight into the occurrence of the transition of the dark energy equation of state and the mechanism behind this transition. Other studies on the w_D crossing -1 can be found in [18].

Most discussions on dark energy rely on the fact that its evolution is independent of other matter fields. Given the unknown nature of both dark energy (DE) and dark matter (DM), which are two major contents of the universe, one might argue that an entirely independent behavior of DE is very special [19]. Studies about the interaction between DE and DM have been carried out in [19, 20, 21, 22]. It was argued that the interaction will influence the perturbation dynamics and could be observable through the lowest multi-poles of the CMB spectrum [20]. Recently, by considering the interaction between DE and DM in the holographic DE model, it was even found that the Hubble scale might be used as the IR cutoff to explain the acceleration of our universe [22]. In this work, we are going to extend the inclusion of interaction between DE and DM into the holographic DE model with the future event horizon as the IR cutoff. The ratio of energy densities can be varied with time. As a result, we find that with the interaction between DE and DM, the model can give an early deceleration and a late time acceleration. In addition, in this holographic model, the appropriate coupling between DE and DM accommodates the transition of the DE equation of state from $w_D > -1$ to $w_D < -1$. This property could serve as an observable feature of the interaction between DE and DM in addition to its influence on the small l CMB spectrum argued in [20].

The total energy density is $\rho = \rho_m + \rho_D$, where ρ_m is the energy density of matter and $\rho_D = 3c^2 R_E^{-2}$ is the dark energy density. Here, the Planck mass is taken as unit. We followed [4] by choosing the future event horizon, $R_E = a \int_a^\infty \frac{dx}{Hx^2}$, as the IR cutoff, where c^2 is a constant. The total energy density satisfies a conservation law. However since we consider the interaction between DE and DM, ρ_m and ρ_D do not satisfy independent conservation laws, they instead satisfy

$$\dot{\rho}_m + 3H\rho_m = Q \quad (1)$$

and

$$\dot{\rho}_D + 3H(1 + w_D)\rho_D = -Q \quad , \quad (2)$$

where w_D is the equation of state of DE, Q denotes the interaction term and can be taken as $Q = 3b^2 H\rho$ with b^2 the coupling constant [22]. This expression for the interaction term was first introduced in the study of the suitable coupling between a quintessence scalar field and a

pressureless cold dark matter field [19]. The choice of the interaction between both components was to get a scaling solution to the coincidence problem such that the universe approaches a stationary stage in which the ratio of dark energy and dark matter becomes a constant. In the context of holographic DE model, this form of interaction was derived from the choice of Hubble scale as the IR cutoff [22].

Taking the ratio of energy densities as $r = \rho_m/\rho_D$, from (1) and (2) we have

$$\dot{r} = 3b^2 H(1+r)^2 + 3Hrw_D \quad . \quad (3)$$

In [22], the IR cutoff was chosen as being the Hubble scale, which leads to a constant r . We choose the future event horizon as the IR cutoff so that r is no longer a constant in the holographic DE model.

Using the Friedmann equation, $\Omega_m + \Omega_D = 1$, where $\Omega_m = \rho_m/(3H^2)$ and $\Omega_D = \rho_D/(3H^2)$, we have $r = (1 - \Omega_D)/\Omega_D$ and $\dot{r} = -\dot{\Omega}_D/\Omega_D^2$. Combining with (3), we get the equation of state of DE,

$$\begin{aligned} w_D &= [-\dot{\Omega}_D/\Omega_D^2 - 3b^2 H(1+r)^2]/(3Hr) \\ &= -\frac{\Omega'_D}{3\Omega_D(1-\Omega_D)} - \frac{b^2}{\Omega_D(1-\Omega_D)} \quad , \end{aligned} \quad (4)$$

where the dot is the derivative with respect to time and the prime is the derivative with respect to $x = \ln a$.

From the Friedmann equation, the future event horizon can be expressed as $R_E = c\sqrt{1+r}/H = a \int_t^\infty dt/a$. Taking the derivative with respect to t and using the definitions of r and \dot{r} as well as equation (4), we arrive at

$$\frac{\Omega'_D}{\Omega_D^2} = (1 - \Omega_D) \left[\frac{1}{\Omega_D} + \frac{2}{c\sqrt{\Omega_D}} - \frac{3b^2}{\Omega_D(1 - \Omega_D)} \right] \quad . \quad (5)$$

Back to (4) we get now

$$w_D = -1/3 - 2\sqrt{\Omega_D}/(3c) - b^2/\Omega_D \quad . \quad (6)$$

Neglecting the interaction between DE and DM, (5) is equivalent to equation (18) of Li's paper in [4]. We will show that the interaction between DE and DM brings rich physics.

The deceleration parameter can be expressed in this model as

$$\begin{aligned} q &= -\ddot{a}a/a^2 = -\dot{H}/H^2 - 1 \\ &= 1/2 - 3b^2/2 - \Omega_D/2 - \Omega_D^{3/2}/c \quad , \end{aligned} \quad (7)$$

where $\dot{H}/H^2 = 3(-1 - w_D - r)\Omega_D/2$.

With eqs (4-7), we are in a position to investigate the evolution of the DE and its influence on the expansion of the universe.

Since the DE plays a more important role in the evolution of the universe with the flow of cosmological time, we require $\Omega'_D > 0$, which leads to

$$b^2 < b_{max}^2 = (1 - \Omega_D)(1 + 2\sqrt{\Omega_D}/c)/3 \quad . \quad (8)$$

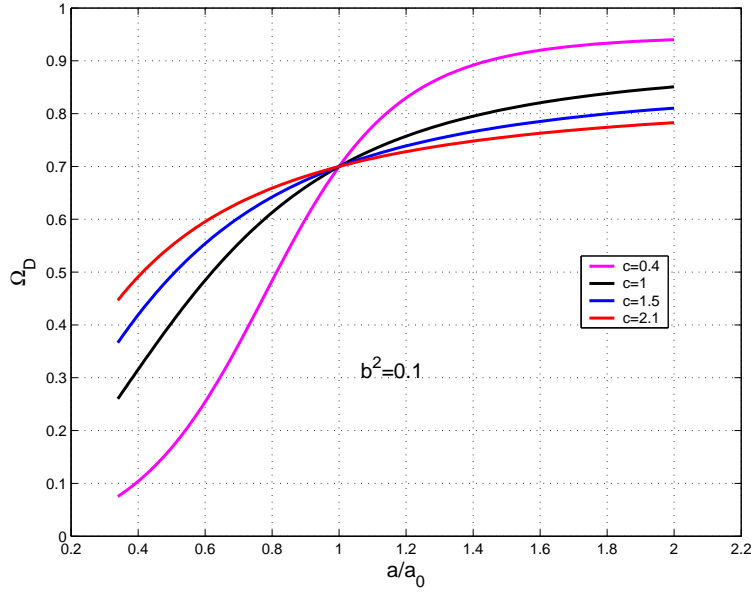


Figure 1: Evolution of the DE for a fixed interaction parameter with DM (b^2) but for different values of the constant c .

The behavior of the DE evolution puts an upper limit on the interaction between DE and DM, which has not been observed in the study of the holographic DE model by taking the Hubble scale as IR cutoff [22]. b_{max}^2 decreases with the increase of c . Choosing two different values of c with $c_1 < c_2$, their corresponding largest allowed values b_{max}^2 are b_{1-max}^2 and b_{2-max}^2 and satisfy $b_{1-max}^2 > b_{2-max}^2$. Taking b_s^2 as a common allowed value of the coupling for c_1 and c_2 cases, it is easy to see that b_s^2 is closer to b_{2-max}^2 than b_{1-max}^2 , which means that in the large c case the same allowed coupling b^2 is stronger than in the small c case.

The dependences of the evolution of DE with respect to the constants b^2, c are shown in Fig. 1 and Fig. 2, respectively. From Fig. 1 we learn that for the fixed coupling between DE and DM within the allowed range (8), the DE starts to be effective earlier when c is larger. Since the same

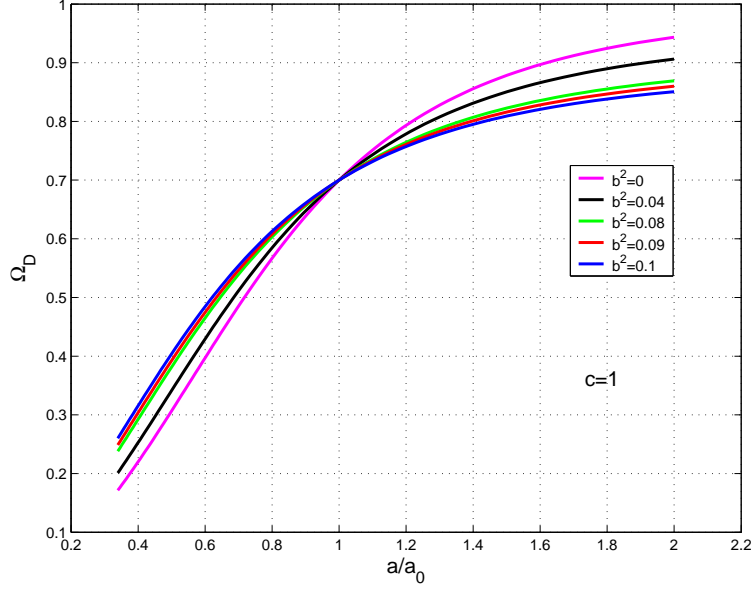


Figure 2: Evolution of the DE with a fixed constant c but different values for the coupling with DM.

b^2 corresponds to a stronger coupling between DE and DM for bigger c cases, Ω_D will tend to a smaller value at late stage when c is bigger due to this stronger coupling.

For a fixed c , the dependence of Ω_D on b^2 is shown in Fig. 2. When b^2 is larger, at early stage, the DE starts to be effective earlier. However at a later stage, since the coupling between DE and DM becomes stronger, Ω_D approaches a smaller value for a larger b^2 .

Including the interaction, our model naturally shows that our universe has an accelerated expansion in the late stage and on the other hand it also displays a deceleration in the early era. In Fig. 3 we show that for the same coupling between DE and DM, the acceleration starts earlier for larger c , since the DE develops earlier for larger c with the same coupling b^2 . In Fig. 4 we show that for the same c , the acceleration starts earlier for larger b^2 . This is also due to the fact that for the same c , DE develops earlier for bigger b^2 as shown in Fig. 2.

We now discuss the equation of state of the DE with the interaction between DE and DM. From (7) we learnt that w_D has a maximum value $w_{D-max} = -1/3 - (b/\sqrt{3}c)^{2/3}$ at $\Omega_{D-cr} = (3b^2c)^{2/3}$. To allow the DE transition as indicated by recent observations with w_D crossing the border -1 and $w_D < -1$ at present stage, we need $b^2 > b_{cr}^2 = 2\Omega_D(1 - \sqrt{\Omega_D}/c)/3$, where b_{cr}^2 is got from $w_D = -1$. Such b_{cr}^2 should be in the reasonable range (8). Thus we get $c < 2\sqrt{\Omega_D}/(3\Omega_D - 1)$. Meanwhile b_{cr}^2 should be positive, which leads to $c > \sqrt{\Omega_D}$. According to the holographic principle, the entropy of the universe is bounded by $S = \pi R_E^2$, where $R_E = c/(H\sqrt{\Omega_D})$. If we require the entropy of the

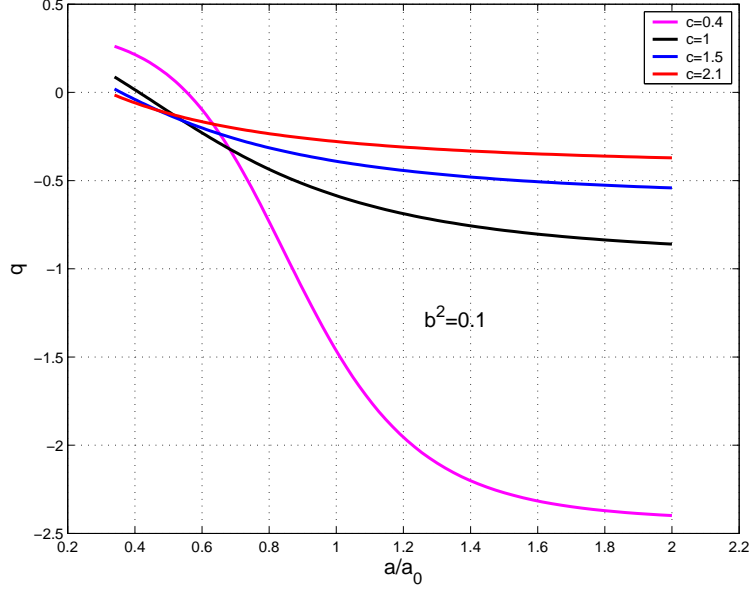


Figure 3: Dependence of the deceleration parameter on the constant c for a fixed coupling between DE and DM.

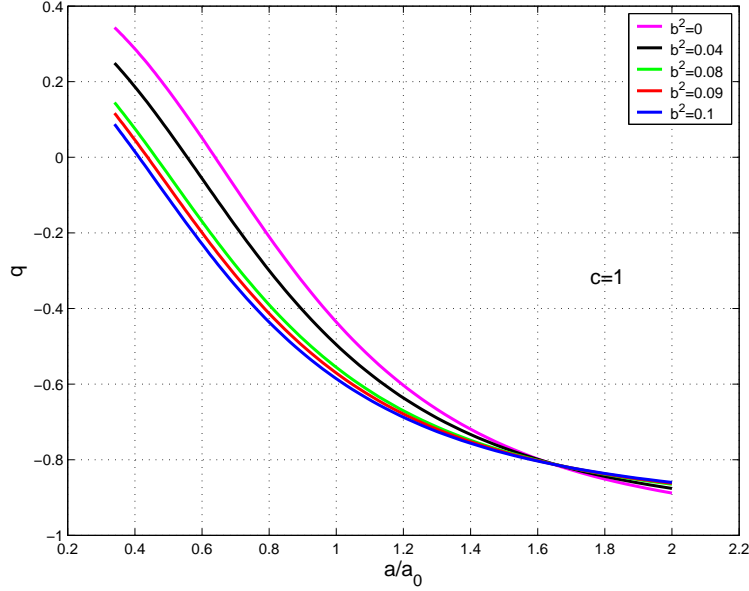


Figure 4: Dependence of the deceleration parameter on the coupling between DE and DM for a fixed constant c .

universe do not decrease, we need $\dot{R}_E = c/\sqrt{\Omega_D} - 1 \geq 0$, thus we get $c \geq \sqrt{\Omega_D}$. Therefore the lower bound on c is exactly the requirement of the second law of thermodynamics discussed in [4]. Since the DE evolves independently of DM in cases ($b^2 = 0$), $w_D < -1$ requires $c < \sqrt{\Omega_D}$, which

violates the second law. Thus in the holographic model with entirely independent behavior between DE and DM, it is impossible to have w_D crossing -1 allowing the phantom energy to exist in the late stage of the universe.

Further examining the function b_{cr}^2 , we see that it has a maximum value $b_{cr-max}^2 = 8c^2/81$ at $\Omega_D = 4c^2/9$. In order to have $w_D < -1$, we need $b^2 > b_{cr}^2$. However the coupling b^2 cannot be arbitrarily large, since when $b^2 > b_{cr-max}^2$ and $w_D < -1$ always. The upper bound on b^2 can also be got by requiring $w_{D-max} > -1$ thus allowing w_D to cross -1 and to stay below -1 later. In order to have $w_D < -1$ with $\Omega_{D0} = 0.7$, such a maximum value b_{cr-max}^2 should appear early when $\Omega_D = 4c^2/9 < 0.7$. Combining constraints on c , we have

$$\sqrt{\Omega_D} < c < 1.255 \quad . \quad (9)$$

Now let us focus on the parameter space of b^2 , which we have already obtained, $b_{cr}^2 < b^2 < 8c^2/81$. Meanwhile taking account of the behavior of the w_D function, we also require $\Omega_{D-cr} < \Omega_{D0}$, which leads to $b^2 < \Omega_{D0}^{3/2}/(3c)$, allowing the w_D transition from $w_D > -1$ to $w_D < -1$ in recent times. Using $\Omega_{D0} = 0.7$, the combined constraint on b^2 reads

$$1.4(1 - \sqrt{0.7}/c)/3 < b^2 < 8c^2/81 \quad , \quad (10)$$

to accommodate w_D crossing -1 .

For a fixed c , the dependence of w_D on b^2 is shown in Fig. 5. We see that for larger b^2 , w_D crosses -1 earlier. For a too small b^2 , w_D crosses -1 too late. Actually these small b^2 are over the lower bound of (10) and are not consistent with the observation.

From the future precise observation of the location of the transition of the w_D from $w_D > -1$ to $w_D < -1$, it is possible to understand the interaction between DE and DM. In the future this could serve as another observable feature of the interaction between DE and DM in addition to the low CMB spectrum discussed in [20].

We have also fitted our model with the golden SN data. We got $\Omega_{m0} = 0.39_{-0.16}^{+0.14}$, $b^2 = 0.00_{-0.00}^{+0.11}$, $c = 0.40_{-0.30}^{+0.75}$ and $\chi^2 = 174.44$. If we fix $c = 1$, we have $\Omega_{m0} = 0.26_{-0.05}^{+0.14}$, $b^2 = 0.00_{-0.00}^{+0.12}$, and $\chi^2 = 177.08$. This shows that our model is consistent with the SN data and the constraint on parameter spaces we discussed are compatible with the observations.

In summary, we have studied the holographic dark energy model with an interaction between DE and DM. Choosing the future event horizon as the IR cutoff, we obtained a generalized model with the time dependent ratio of energy densities which cannot be realized just by adopting the Hubble scale as the IR cutoff. With the interaction between DE and DM, a richer physics arises.

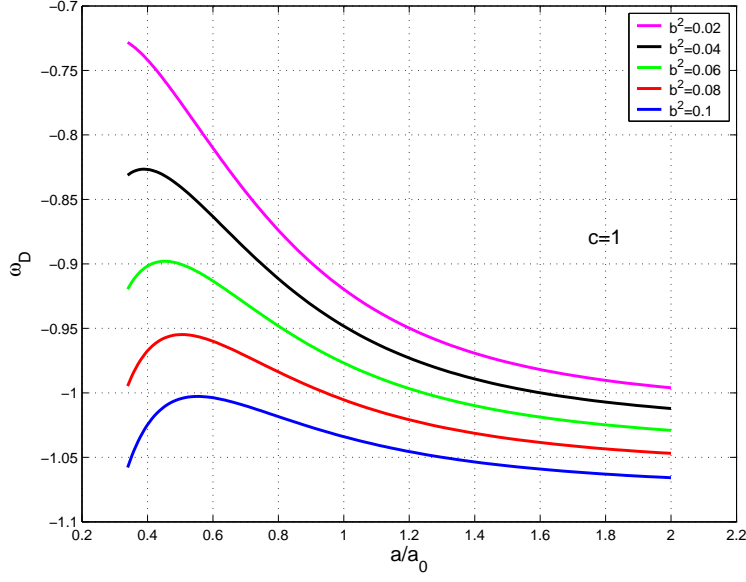


Figure 5: Behavior of the equation of state of DE for different couplings between DE and DM.

In addition to showing the comprehensive history of the evolution of our universe from the early deceleration to late acceleration, we have also found that the interaction between DE and DM can accommodate the transition of the equation of state of DE from $w_D > -1$ to $w_D < -1$. We have constrained the parameter space of our model to explain the observations. We argued that in addition to some observational feature for small l CMB spectrum [20], the interaction between DE and DM could also be observed by the w_D crossing -1 behavior in the future. Comparison to the golden SN data has been made, and we showed that our model is also consistent with such observations.

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